

Shell-model calculations for the energy levels of the $N = 50$ isotones with $A = 80-87$

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The detailed features of the calculated energy-level schemes and of the single-particle, orbit-occupancy properties of the low-lying levels of the $N = 50$ isotones ^{80}Zn , ^{81}Ga , ^{82}Ge , ^{83}As , ^{84}Se , ^{85}Br , ^{86}Kr , and ^{87}Rb are presented and discussed. These results are obtained with a new effective Hamiltonian operator obtained empirically from an iterative fit to experimental energies taken from all experimentally studied ($A = 82-96$) $N = 50$ nuclei. The model space for the calculations consists of active $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0g_{9/2}$ proton orbits relative to a nominal ^{78}Ni core. This space is truncated internally by restricting the number of particles excited from the negative-parity orbits into the $g_{9/2}$ orbit to be no greater than four. The typical structures predicted for these lighter $N = 50$ isotones are found to be dominated by well-mixed combinations of fp -orbit configurations, with the $g_{9/2}$ orbit playing a minor role in all but a few special cases. The model energy-level spectra are compared with existing experimental information, as are calculated spectroscopic factors for single-proton stripping and pickup reactions.

I. INTRODUCTION

We have created an effective model Hamiltonian for shell-model calculations of the structure of the $N = 50$ isotones from ^{79}Cu to ^{100}Sn .¹ The model space utilized consists of the $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0g_{9/2}$ proton orbits, with the only truncation being that no more than four particles are allowed to be excited from the fp subspace to the g subspace relative to the lowest-order configuration. Comparisons of results obtained in the untruncated space of these orbits with the truncated results show only insignificant differences. The present calculations replicate,^{1,2} with relatively minor improvements, the several successful calculations for nuclei above ^{90}Zr which utilize the smaller $p_{1/2}$ - $g_{9/2}$ proton space. In this context, our principal focus is on the nuclei at and below ^{90}Zr , for which the two-orbit space is, *a priori*, inadequate. In this paper we present results of calculated level energies and spectroscopic factors for single-proton transfer in comparison with existing experimental data for the $N = 50$ nuclei up through ^{87}Rb . From these comparisons we hope to learn more about the structure of these lighter $N = 50$ nuclei, to evaluate various aspects of the model's capability, and to determine fruitful directions for further experimental research and further refinement of the model.

II. RESULTS AND DISCUSSION

A. General remarks on experimental-theoretical comparisons

There are two levels of difficulty in determining the experimental energy level spectra with which the predicted energies of $N = 50$ levels should be compared. The pri-

mary problem is just to count every actual experimental level once and only once. This is a nontrivial task for all the nuclei considered here, particularly for the lighter isotones.³ The farthest-from-stability systems are populated by only one or two nuclear processes and these processes typically are both selective and experimentally arduous. Thus, not all levels may be observed, or be observed with adequate statistics to be unambiguously established. Even in the more thoroughly studied nuclei nearer to ^{88}Sr , the combination of the selectivity of the reaction mechanisms and the limitations of the energy resolution and counting statistics of the existing data make it impossible to say with confidence whether all levels have been observed and/or whether a level observed at approximately the same excitation energy in two different reactions is the same level or two different levels.

The second level of difficulty concerns well-established experimental levels which, on the basis of spin-parity-energy values alone or because of additional experimental signatures, do not appear to correspond to levels generated within the model space. Ideally, such levels would not occur at lower excitation energies. But, even when such intruder states are present in the low-excitation part of the spectra, it has been found that they can coexist with the resident states of the space without causing serious perturbations. It is, however, important to identify these intruders wherever possible so as to avoid biasing the model parameters in an attempt to fit inappropriate data. Intruder states for our present model space would be expected to exhibit evidence either of active $0f_{7/2}$ or $2s$, $1d$, $0g_{7/2}$ proton configurations or of any neutron configuration other than the $N = 50$ closed shell. The key signatures which would identify such intruder states would be appreciable spectroscopic factors for single-proton transfer of the wrong j values or any significant

TABLE I. States of ^{80}Zn : Calculated excitation energies (th) and occupation numbers. The index n refers to the first (1), second (2), etc., state of a given value of J^π .

J^π	n	Energy (keV)	Occupation of orbit $j^\pi (\times 100)$			
		th	$j^\pi = \frac{5}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^+$
0^+	1	0	160	26	13	1
2^+	1	1276	187	6	7	0
4^+	1	2382	176	24	0	0
2^+	2	2888	88	102	10	0
1^+	1	2900	100	100	0	0
3^+	1	3112	100	75	25	0
4^+	2	3362	124	76	0	0
0^+	2	3943	32	163	1	4

single-neutron or two-neutron transfer strength to an excited state.

In the following sections we present the calculated energy-level spectra for the nuclei under study together with experimental energy level data as available. Where single-proton transfer experiments can populate a nucleus we also show calculated and measured spectroscopic factors. In other cases we present calculated occupation numbers for the various states.

B. Levels for ^{80}Zn

There is no experimental information about ^{80}Zn . The calculated spectrum, as presented in Table I, is characterized by a $0^+, 2^+, 4^+$ sequence for the lowest three states, these states arising predominately from the $(f_{5/2})^2$ configuration. The next 2^+ and 4^+ states and the 1^+ and 3^+ states arise from the $(f_{5/2})^1(p_{3/2})^1$ configuration. These are the simple consequences of having the $f_{5/2}$ orbit being the most tightly bound orbit in our model Hamiltonian, followed by the $p_{3/2}$ orbit.¹

C. Levels of ^{81}Ga

The only experimental information³ about ^{81}Ga is the inferred ground-state spin-parity of $\frac{5}{2}^-$. The predicted

spectrum of ^{81}Ga presented in Table II shows a $\frac{3}{2}^-$ and $\frac{5}{2}^-$ ground-state doublet, with the $\frac{5}{2}^-$ state coming 64 keV above the $\frac{3}{2}^-$ state. It would be of obvious interest to establish experimentally the excitation energies of the lowest several states of ^{81}Ga . The occupation numbers show that the lowest $\frac{3}{2}^-$ state is basically a seniority-three $(f_{5/2})^3$ state. The $p_{3/2}$ one-quasiparticle state comes at 679 keV. Likewise, the lowest $\frac{1}{2}^-$ state is predicted to arise from the $(f_{5/2})^2(p_{3/2})^1$ configuration, not from the $p_{1/2}$ one-quasiparticle configuration, which shows up only at 2.1 MeV.

D. Levels of ^{82}Ge

Our only experimental information on ^{82}Ge comes from study^{4,5} of the γ rays following the β decay of $^{82}\text{Ga}(1^+)$. The eight energy levels inferred from γ - γ coincidence measurements⁵ are listed in Table III along with the model's calculated spectrum. The experimental data provide only wide limits on spin-parity assignments. The experimental levels are correlated with the model levels simply by listing them in ascending order. The match-ups therefore have no deep justification, but they

TABLE II. States of ^{81}Ga : Calculated excitation energies (th) and occupation numbers. The index n refers to the first (1), second (2), state of a given value of J^π .

J^π	n	Energy (keV)	Occupation of orbit $j^\pi (\times 100)$			
		th	$j^\pi = \frac{5}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^+$
$\frac{3}{2}^-$	1	0	274	7	19	0
$\frac{5}{2}^-$	1	64	256	29	14	1
$\frac{3}{2}^-$	2	679	186	104	11	0
$\frac{1}{2}^-$	1	1279	172	118	11	0
$\frac{5}{2}^-$	2	1368	185	96	18	1
$\frac{3}{2}^-$	3	1897	181	102	16	0
$\frac{5}{2}^-$	3	2009	159	135	5	1
$\frac{7}{2}^-$	2	2134	183	18	99	0
$\frac{5}{2}^-$	4	3101	194	29	78	0
$\frac{3}{2}^-$	4	3105	119	155	25	1
$\frac{1}{2}^-$	3	3887	100	152	48	1
$\frac{9}{2}^+$	1	4243	185	9	5	101

TABLE III. States of ^{82}Ge : Calculated (th) and experimental (exp) excitation energies and calculated occupation numbers. The index n refers to the first (1), second (2), etc., state of a given value of J^π .

J^π	n	Energy (keV)		$j^\pi = \frac{5}{2}^-$	Occupation of orbit $j^\pi (\times 100)$		
		th	exp ^a		$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^+$
0^+	1	0	0	295	74	28	3
2^+	1	1337	1348	332	46	21	1
2^+	2	1974	2215	265	115	19	1
3^+	1	2328	2287	278	101	20	1
0^+	2	2339	2334	276	118	6	0
1^+	1	2368	2703	284	102	14	0
2^+	3	2566	2713	275	94	30	1
4^+	1	2629		313	68	13	1
4^+	2	2778		295	87	17	1
3^+	2	2961		272	101	20	1
2^+	4	3049	3258	245	92	60	4

^aReference 5.

at least do not seem to violate any simple aspects of the observed features; e.g., the assumed 3_1^+ and 0_2^+ states do not gamma decay to the 0^+ ground state. Beyond this point, however, the correspondences are arbitrary and are made principally to illustrate that the observed and calculated level densities are consistent. We note in this context that 4^+ levels would be unlikely to have appeared in the experimental observations.

The model occupation numbers suggest that the $(f_{5/2})_J^4$ configuration dominates the ground and first excited states, with the $p_{3/2}$ orbit contributing the principal admixtures. The second and third 2^+ states, along with the 1^+ , 3^+ , and 0_2^+ states, are constructed with significantly greater $p_{3/2}$ occupancy, at the expense of the $f_{5/2}$ orbit. We conclude from the comparisons made in Table III that the calculated level density is consistent

TABLE IV. States of ^{83}As : Calculated (th) and experimental (exp) excitation energies and calculated occupation numbers. The index n refers to the first (1), second (2), etc., state of a given value of J^π .

J^π	n	Energy (keV)		$j^\pi = \frac{5}{2}^-$	Occupation of orbit $j^\pi (\times 100)$		
		th	exp ^a		$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^+$
$\frac{5}{2}^-$	1	0	0	382	86	28	4
$\frac{3}{2}^-$	1	189	306	354	119	26	1
$\frac{1}{2}^-$	1	353	712	346	130	24	1
$\frac{3}{2}^-$	2	589	1194	310	144	40	6
$\frac{7}{2}^-$	1	990	1196	358	110	31	1
$\frac{5}{2}^-$	2	1115	1257	371	103	24	1
$\frac{3}{2}^-$	3	1253	1330	317	158	21	4
$\frac{1}{2}^-$	2	1382	1415	308	121	69	2
$\frac{5}{2}^-$	3	1455	1435	322	148	26	4
$\frac{9}{2}^-$	1	1643	1525	348	124	26	2
$\frac{3}{2}^-$	4	1845	1543	299	151	47	2
$\frac{1}{2}^-$	3	1857	1804	297	141	59	2
$\frac{7}{2}^-$	2	1879	1977	337	140	22	1
$\frac{9}{2}^-$	2	1985	2233	310	140	49	1
$\frac{7}{2}^-$	3	2013		296	166	36	1
$\frac{5}{2}^-$	4	2038		306	139	53	1
$\frac{5}{2}^-$	5	2279		315	163	20	2
$\frac{3}{2}^-$	5	2357		285	163	48	4
$\frac{5}{2}^-$	6	2704		289	182	26	3
$\frac{7}{2}^-$	4	2719		337	115	46	2
$\frac{9}{2}^+$	1	2752		275	109	15	101

^aReference 6.

with the current stage of experimental results, although the predicted energies of some of the excited states come too low relative to their presumed counterparts.

E. Levels of ^{83}As

As with ^{82}Ge , the only experimental information available about the levels of ^{83}As comes from observation of the γ rays following β decay, in this case from $^{83}\text{Ge}(\frac{5}{2}^+(?))$. From γ singles and γ - γ coincidence measurements,⁶ 14 levels are established in the range of excitation energy extending up to 2300 keV. These levels and the model levels predicted to lie below 2.8 MeV excitation energy are listed in Table IV. (Another 14 experimental levels are established⁶ in the region of excitation energy above 3 MeV.) As also was the case for ^{82}Ge , experiment does not yield firm information on spin-parity assignments for the levels of ^{83}As other than the limits suggested by the β decay $\log ft$ values and the observation or nonobservation of γ decay branches. We have assumed J^π values of $\frac{5}{2}^-$ for the ground state of ^{83}As and $\frac{3}{2}^-$ for the first excited state, even though the systematics from the lighter As isotopes⁷ would suggest $J^\pi = \frac{3}{2}^-$ for the ground state. The model wave functions for each of these two states reveal that they are the one-quasiparticle

$f_{5/2}$ and $p_{3/2}$ states, respectively, a feature which will reappear in ^{85}Br and ^{87}Rb . In the heavier systems the characters of the levels are experimentally confirmed via single-proton transfer reaction data.

In attempting to correlate observed levels with the model predictions for ^{83}As we are limited both by the lack of spin-parity information and the lack of certainty as to whether all levels existing below 2.3 MeV excitation energy have been observed. If we assume that levels with spins $\frac{1}{2}^+ - \frac{9}{2}^+$ can be observed, then more levels are predicted in this range than have been observed. However, if it is assumed that the β decay is more restrictive, as has been assumed for the analogous ^{85}Br decay, then we predict fewer levels than experiment has already revealed. A question of particular interest arises below 1 MeV, where we predict the second $\frac{3}{2}^-$ and the first $\frac{1}{2}^-$ levels to occur, while only one level in this region, at 712 keV, is observed above the ground-state doublet. We note, as can be inferred from Table IV, that the lowest $\frac{1}{2}^-$ state in the model is not the $p_{1/2}$ single-quasiparticle state.

The most straightforward correlation of model and observed levels for ^{83}As is obtained by assuming that no low-lying levels are missed in the experiment and by matching up experimental and theoretical levels one by one in ascending order. This arbitrary approach is used

TABLE V. States of ^{84}Se : Calculated (th) and experimental (exp) excitation energies and calculated occupation numbers. The index n refers to the first (1), second (2), etc. state of a given value of J^π .

J^π	n	Energy (keV)		Occupation of orbit $j^\pi (\times 100)$			
		th	exp	$j^\pi = \frac{5}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^+$
0^+	1	0	$0^{a,b,c}$	415	136	42	7
2^+	1	1672	$1455^{a,b,c}$	378	184	31	8
0^+	2	2122	1967^c	405	158	32	4
1^-			$2097^{c,d,e}$				
3^+	1	2125		437	122	39	2
2^+	2	2150	2122^a	428	137	32	3
0^+			$2247^{b,c,e,f}$				
0^+			$2655^{b,c,e,f}$				
1^+	1	2706	2700^a	418	157	22	2
0^+			$2716^{c,d,e}$				
4^+	1	2837		417	144	36	3
2^+	3	2801		359	182	54	4
4^+	2	2837		389	178	28	5
0^+	3	2862	2740^c	343	191	57	8
2^+			$2984^{b,c,e,f}$				
3^+	2	2980		380	163	55	2
2^+	4	3107	3022^c	412	136	48	4
3^+	3	3353		345	222	28	5
1^+	2	3482	3299^a	362	150	84	4
0^+	4	3548		400	117	74	9
1^+	3	3650		351	267	26	6
2^+	5	3657	$3542^{a,b,c}$	368	157	71	4
4^+	3	3691		365	201	25	9
			$3693^{b,c,e,f}$				

^aReference 8.

^bReference 9.

^cReference 10.

^dWeak in (t, p) .

^eAssumed to be an intruder relative to model space.

^fStrong in (t, p) .

to produce the correspondence shown in Table IV. More detailed considerations might suggest switching some correspondences or omitting some altogether. However, at this stage of experimental knowledge it is not clear that much would thereby be gained. The salient facts which can be established at this point are that the model spectrum in the region up to 2 MeV excitation energy contains approximately the same number of levels as is observed in this region experimentally and that the predicted energies of the lowest few excited states are significantly too low with respect to experiment, in particular if the current experimental spectrum reflects all levels which actually exist below 1200 keV.

F. Levels of ^{84}Se

There is experimental information on ^{84}Se both from the observation of the γ rays following the β decay⁸ of $^{84}\text{As}(1^-)$ and from the $^{82}\text{Se}(t,p)$ reaction.^{9,10} The model levels predicted to fall within the first 3.7 MeV of excitation energy are listed in Table V along with the energies of levels observed experimentally within this same energy range. The occupation numbers reveal the effects of the $f_{5/2}$ shell closure. While the $p_{3/2}$ orbit is appreciably occupied for all states, the value is below average for the 0^+ ground state, for which $(f_{5/2})^6$ can be considered the "parent" configuration, and significantly above average for the first 2^+ first excited state, for which the $(f_{5/2})^5(p_{3/2})^1$ configuration can be considered the parent.

The γ rays following the β decay into ^{84}Se yield evidence for five or six levels below 3.7 MeV excitation energy. No level except the first excited state is recorded as having a ground-state decay branch. This seems anomalous, since several of these levels should have J^π of 1^+ or 2^+ . (See, for example, the case of ^{82}Ge , Ref. 5.) Only the ground, first excited, and 3.541 MeV excited states are identified both in the β - γ and the (t,p) studies. Thus, counting the two or three states observed in the γ -ray measurements (the 3298 keV level is only tentatively identified) but not seen in (t,p) , together with the 12 levels identified in this same region in the latest (t,p) study,¹⁰ there are as many as 15 levels experimentally observed below 3.7 MeV excitation energy, while eighteen are predicted in our model.

The relationships between experimental and model results are more confusing than these sums alone would indicate, however. There are, in particular, more 0^+ levels identified experimentally than are predicted and several 3^+ and 4^+ states predicted which have not been experimentally identified. Some of the surplus observed levels can be attributed to core-breaking neutron excitations by virtue of the strength with which they are populated in the (t,p) reaction. On this basis we would assign the 0^+ levels observed at 2247 and 2655 keV and the 2^+ level observed at 2984 keV as neutron-excitation intruders relative to our model space. These levels appear to have two neutron particle-two neutron hole structures relative to a closed $N=50$ core. Most of the remaining states in the low-lying part of the (t,p) spectrum can be identified with model states as indicated in Table V. The exceptions are the suggested 1^- state observed at 2097 keV and the third excited 0^+ level, observed at 2716 keV. The

(t,p) spectrum does not show evidence for the 2122, 2700, and 3298 keV states inferred from the γ ray data, unless one identifies the 2716 keV $0^+(?)$ level assigned¹⁰ in (t,p) with the 2700 keV level, which we have matched with the model 1^+ state in Table V.

The conclusions which can be drawn about ^{84}Se are as ambiguous, or more so, than those possible for the lighter nuclei, in spite of the new experimental avenue open for its study. The (t,p) data highlight the role of neutron-excitation intruders, which appear at rather low excitation energy. At the same time, the (t,p) results suggest a 1^- state at 2 MeV which is highly anomalous, and apparently fail, as does the β decay, to detect the 3^+ and 4^+ states predicted by the model, and maybe even miss a predicted 2^+ state. It is difficult to evaluate how theory needs to be improved for ^{84}Se without better experimental guidance.

G. levels of ^{85}Br

The level structure of ^{85}Br has been studied in experiments which use the single-proton pickup reaction^{11,12} on ^{86}Kr and the β decay, followed by γ decay,¹³ of the $\frac{5}{2}^+$ ground state of ^{85}Se . In accord with the authors of Ref. 13, we assume that the β decay populates levels which differ from the parent by at most one unit of angular momentum. Thus, we assume that the β decay identifies all levels of $J=\frac{3}{2}, \frac{5}{2},$ and $\frac{7}{2}$ up to 2.3 MeV and does not populate states with values of $J=\frac{1}{2}$ or of $\frac{9}{2}$ and greater. We assume that the more recent pickup experiment,¹² which was characterized by an energy resolution of only 220 keV, populated levels allowed within the model space restrictions in this region of excitation, namely with $J^\pi=\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-,$ and $\frac{9}{2}^+$, and correctly identified the l values associated with the proton transfer to such states. This experiment also measured the values of the transferred j as well as of l , but the j values reported for weakly populated excited states do not appear to be conclusive in every instance and some of our suggested correspondences disagree with them.

We present the predicted spectrum of ^{85}Br in comparison with these experimental results in Table VI along with the predicted and observed spectroscopic factors for pickup of a proton from ^{86}Kr . The ground and first excited states are securely identified as $J^\pi=\frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively. The predicted spectroscopic factors agree with the experimental values and establish these two levels as the $p_{3/2}$ and $f_{5/2}$ one-quasihole states, respectively, relative to an ^{86}Kr ground state wave function that is predominantly $(f_{5/2})^6(p_{3/2})^2$.

The β - γ decay data¹³ establish two levels near 1 MeV excitation, at 955 and 1191 keV. (The 941 keV level noted in Refs. 3 and 12 is attributed to a crossover γ transition in Ref. 13.) We predict two levels at this energy (1013 and 1021 keV) which have spins $(\frac{3}{2}^-$ and $\frac{5}{2}^-)$ consistent with the β decay spin restrictions. We assume these associations in Table VI. We also predict a $\frac{1}{2}^-$ state at 745 keV. We associate this model $\frac{1}{2}^-$ state with the 0.9 MeV state observed in the proton pickup experiment with $l=1$. This is in contradiction with the associated $J=\frac{3}{2}$ assignment for this state in Ref. 12 but pro-

duces consistency between the predicted and observed $l=1$ spectroscopic factors. The pickup data identify another $l=1$ level, this is one with $J=\frac{1}{2}$, at 1.2 MeV excitation. This level we identify with the second model $J=\frac{1}{2}^-$ state, predicted to lie at 1262 keV. The level observed in the β - γ data at 1191 keV, and assumed above to be the second model $J=\frac{3}{2}^-$ state, may or may not contribute to the observed pickup cross sections for the 1.2 MeV group, but these latter data should be dominated by the second model $\frac{1}{2}^-$ level on the basis of its predicted spectroscopic factor, which is in good agreement with the value extracted for the 1.2 MeV group.

As noted above, we choose to correlate the second model $\frac{5}{2}^-$ state, predicted to occur at 1013 keV, with the 955 keV level observed in the β - γ study. This model state is predicted to have a modest $l=3$ spectroscopic factor, the nonobservation of which, at the level of about 5% of the strength of the first excited state, is on the verge of being a problem for this assignment. A $J=\frac{7}{2}^-$ state is predicted to lie at 1239 keV. We associate this state with the level observed at 1426 keV in the β - γ experiment. The $\frac{9}{2}^-$ state predicted to lie at 1299 keV would not have been observed either in β decay or proton pickup under our assumptions. We associate levels observed between 1.5 and 2.0 MeV in the β - γ study with

model states as shown in Table VI, using as constraints that the $\frac{9}{2}^-$ and $\frac{9}{2}^+$ model states should not be observed experimentally and that the cluster of strength observed in (d, He^3) at 1.79 MeV excitation energy and assigned to have two l values in Ref. 12 contains the 1724 and 1795 keV levels observed in the β - γ study. The remaining observed levels are assigned to the model states in order of ascending excitation energy.

The two $(d, {}^3\text{He})$ studies are in disagreement, as is indicated in Table VI, about the nature of the 1.79 MeV strength, but our prediction of significant $l=1$ and $l=3$ strength at about this energy is in reasonable agreement with the values of Ref. 12. There can be some question as to whether the fourth model $\frac{5}{2}^-$ state, with its spectroscopic factor of 0.23, is counted into the experimental strength of the 1.79 MeV cluster if our assumption that it corresponds to the 1859 keV level observed in the β - γ study is correct.

The two $(d, {}^3\text{He})$ studies also disagree about the nature of the 2.3 MeV group, which Ref. 11 assigns as $l=4$ and Ref. 12 assigns as $l=1$. Our model does not predict any appreciable $l=1$ strength in this region. On the other hand, if the 2.31 MeV level is $\frac{9}{2}^+$ then our predicted energy for this state is clearly too low.

In summary, the model predictions for energies and

TABLE VI. States of ${}^{85}\text{Br}$: Calculated (th) and experimental (exp) values of excitation energies and of spectroscopic factors for single proton pickup from ${}^{86}\text{Kr}$.

J^π	n	Energy (keV)		Pickup $S(j) \times 100$		
		th	exp	th	exp ^b	exp ^c
$\frac{3}{2}^-$	1	0	0 ^{a,b,c}	186	152	170
$\frac{5}{2}^-$	1	125	345 ^{a,b,c}	412	454	446
$\frac{1}{2}^-$	1	745	900 ^b	13	10	
$\frac{5}{2}^-$	2	1013	955 ^a	28		
$\frac{3}{2}^-$	2	1021	1191 ^a	5		
$\frac{7}{2}^-$	1	1239	1427 ^a			
$\frac{1}{2}^-$	2	1262	1200 ^{b,c}	38	39	73
$\frac{9}{2}^-$	1	1299				
$\frac{3}{2}^-$	3	1602	1795 ^{a,b,c}	21	9	40
$\frac{5}{2}^-$	3	1657	1724 ^{a,b}	35	52	
$\frac{7}{2}^-$	2	1886	1553 ^a			
$\frac{9}{2}^+$	1	2020	2310 ^c	10		112
$\frac{5}{2}^-$	4	2138	1859 ^a	23		
$\frac{3}{2}^-$	4	2170	1943 ^a	1		
$\frac{1}{2}^-$	3	2188		2		
$\frac{3}{2}^-$	5	2338	2310 ^b	1	25	
$\frac{1}{2}^-$	4	2463		0		
$\frac{5}{2}^-$	5	2525				

^aReference 13.

^bReference 12.

^cReference 11.

spectroscopic factors for ^{85}Br explain most of the features observed in the ($d, ^3\text{He}$) and β decay studies. The ambiguities in the experimental situation preclude any rigorous conclusions, but it is possible that the predicted level density is approximately correct; i.e., that there are no observed levels which are clearly intruders. The predicted ratio of $f_{5/2}$ to $p_{3/2}$ spectroscopic strength is confirmed experimentally, as is the prediction that approximately one unit of strength is spread from the ground-state doublet into the 1–2 MeV region of excitation energy.

H. Levels of ^{86}Kr

The levels of ^{86}Kr have been studied experimentally with β decay followed by observations of the resulting γ decay,¹⁴ by proton pickup¹⁵ from ^{87}Rb , by the (p, p) reaction,¹⁶ and by the (t, p) reaction.¹⁷ The model levels calculated to fall below 4 MeV excitation energy are listed in Table VII together with a composite experimental spectrum drawn from the above data. Many of the low-lying levels are observed in several different reactions and have reasonably secure spin-parity assignments.

The first three levels of ^{86}Kr , $J^\pi=0^+$, 2^+ , and 4^+ , are securely identified experimentally and their observed energies are well reproduced by the model eigenvalues. The measured spectroscopic factors indicate strong $l=1$ tran-

sitions to the 0^+ and 2^+ levels from the $J^\pi=\frac{3}{2}^-$ ground state of ^{87}Rb , and a strong $l=3$ pickup to the 4^+ level. The model predictions agree with the experimental values to within the conventionally accepted uncertainties in experimentally based spectroscopic factors.

Above these lowest three levels, a 2^+ state at 2.35 MeV and a 0^+ state at 2.72 MeV are securely assigned. In addition, levels are identified at 2.85, 2.93, 3.01, and 3.10 MeV in several different processes. The 3.10 MeV level most probably has $J^\pi=3^-$, the first 3^- state experimentally identified as we have progressed upward in mass along the $N=50$ chain. The lowest model 3^- is found at 3.8 MeV. The failure of shell-model calculations to produce the lowest collective 3^- level in a nucleus at a low enough energy seems to be congenital to (at least) $0\hbar\omega$ identical-particle model spaces, which incorporate only one "different-parity" orbit.

Along with the previously mentioned lowest 4^+ state, the model predicts three 2^+ states and one state each with $J^\pi=0^+$, 1^+ , and 3^+ between 2 and 3 MeV of excitation. Thus, we need to correlate these six additional model states with the five experimentally identified levels between the 4_1^+ and the 3_1^- levels. Two of the observed states have assigned J^π values, 2^+ at 2.35 MeV and 0^+ at 2.72 MeV, as noted above. The 0_2^+ levels in the model and experimental spectra seem to match easily. There are two experimental 2^+ candidates, at 2.35 and 2.85

TABLE VII. States of ^{86}Kr : Calculated (th) and experimental (exp) values of excitation energies and of spectroscopic factors for single proton pickup from ^{187}Rb .

J^π	n	Energy (keV)		$J^\pi=\frac{1}{2}^-$	Pickup $S(j)\times 100$				$L=1$	exp ^b	
		th	exp		th	$\frac{3}{2}^-$	$\frac{5}{2}^-$	$\frac{9}{2}^+$		3	4
0^+	1	0	$0^{a,b,c,d}$		39				53		
2^+	1	1521	$1565^{a,b,c,d}$	1	103	23			88		
4^+	1	2484	$2240^{b,c,d}$			168				178	
1^+	1	2562	$3010^{b,c}$	0	2	63				58	
0^+	2	2572	$2720^{b,c,d}$		6				14		
2^+	2	2600		2	2	0					
3^+	1	2751	$2926^{a,b,c}$		0	95				203	
2^+	3	2919	$2350^{a,b,c,d}$	0	35	43			61		
2^+	4	2949	$2850^{a,b,c}$	17	30	9			44		
0^+	3	3166	$3534^{b,c,d,e}$		7				22		
3^+	2	3449			0	35					
2^+	5	3499	$3322^{b,c}$	4	10	15			12		
			3575^c								
			$3809^{c,d}$								
1^+	2	3627	$3783^{b,c}$	1	2	4			6		
4^+	2	3646				0					
3^-	1	3803	$3100^{a,b,c,d}$					4			4
2^+	6	3809		2	0	7					
5^-	1	3919	$3938^{b,c,d}$								
			$4037^{b,c,d}$								(12)
			$4110^{c,d,e}$								4

^aReferences 14.

^bReferences 15.

^cReferences 16.

^dReferences 17.

^eStrong in (t, p).

MeV excitation, relative to the three model 2^+ states, at 2.60, 2.92, and 2.95 MeV. The lower of these experimental levels is assigned 2^+ in (t, p) . Both have $l=1$ spectroscopic factors in proton pickup which are comparable in magnitude to that of the first observed 2^+ level. The lowest of the three model 2^+ states in question has a small predicted $p_{3/2}$ spectroscopic factor, while the second and third are predicted to have considerable $l=1$ strength. All of the three predicted energies are significantly higher than the 2.35 MeV 2^+ experimental level; hence good energy agreement for this state is impossible. Thus, it seems most appropriate to base the association on spectroscopic factors and match the 2.35 and 2.85 MeV levels with the 2.92 and 2.95 MeV model

2^+ states, leaving the 2.60 MeV model 2^+ unassociated with an experimentally observed level.

The observed levels at 2.92 and 3.01 MeV excitation energies have significant $l=3$ spectroscopic factors. On this basis it seems clear that they should be associated with the model 1_1^+ and 3_1^+ states, predicted at 2.56 and 2.75 MeV, respectively, whose leading configurations consist of an $f_{5/2}$ hole coupled to the $p_{3/2}$ hole that characterizes the ^{87}Rb ground state. There is little basis from energetics for one to choose which is the 1^+ and which is the 3^+ . We suggest that the 3.01 MeV observed level has $J^\pi=1^+$ and the 2.92 MeV level $J^\pi=3^+$ on the grounds that this yields better agreement with the model in so far as relative $l=3$ spectroscopic factors is con-

TABLE VIII. States of ^{87}Rb : Calculated (th) and experimental (exp) values of excitation energies of spectroscopic factors for single proton pickup from ^{88}Sr and stripping from ^{86}Kr .

J^π	n	Energy (keV)		L^a	L^c	Stripping $S(j) \times 100$		th	Pickup $S(j) \times 100$		exp ^g
		th	exp			th	exp ^c		exp ^d	exp ^f	
$\frac{3}{2}^-$	1	0	0 ^{a,b,c,d,e}		1	158	136	307	310	290	340
$\frac{5}{2}^-$	1	483	403 ^{a,b,c,d,e}	2	3	62	118	493	530	510	630
$\frac{1}{2}^-$	1	764	846 ^{a,b,c,d,e}	2	1	136	204	54	50	40	50
$\frac{5}{2}^-$	2	1162	1389 ^{a,e}	2		10		29			
$\frac{9}{2}^+$	1	1437	1578 ^{a,b,c,d,e}	3	4	699	987	21			110
$\frac{3}{2}^-$	2	1620	1463 ^{a,c,e}	2	1	4	3	6			
$\frac{9}{2}^-$	1	1957									
$\frac{1}{2}^-$	2	1965	1740 ^{a,b,c,e}	2		1		1			
$\frac{7}{2}^+$	1	2410	2415 ^{a,b,c}	3							
$\frac{7}{2}^-$	1	2462	2013 ^a	2							
$\frac{11}{2}^+$	1	2615	2379 ^{a,b}	5							
$\frac{5}{2}^-$	3	2618	2284 ^a	2		3		2			
$\frac{9}{2}^+$	2	2629	2732 ^{a,c}	3	4	75	14	3			
$\frac{3}{2}^-$	3	2694	2398 ^{a,c}	2	1	5	17	0			
$\frac{5}{2}^+$	1	2792	2813 ^{a,b,c}	3	2						
$\frac{7}{2}^+$	2	2891	2555 ^{a,b,c}	3	2						
$\frac{3}{2}^-$	4	3022	3340 ^{a,c}		1	0	6	6			
$\frac{13}{2}^+$	1	3051	3002 ^a	5							
$\frac{1}{2}^-$	3	3076				4		6			
			3058 ^{a,b,c}		0						
$\frac{5}{2}^-$	4	3169	3356 ^a			10		22			
$\frac{11}{2}^+$	2	3208	3099 ^a	5							
$\frac{5}{2}^+$	2	3239	3311 ^{a,b,c}	3	2						
$\frac{13}{2}^+$	2	3300	2962 ^{a,b,c}	3+5							
$\frac{9}{2}^+$	3	3371	2977 ^{a,c}	3+5	4	12	13	0			

^aReference 18.

^bReference 19.

^cReference 21.

^dReference 22.

^eReference 25.

^fReference 23.

^gReference 24.

cerned. The model and measured $f_{5/2}$ spectroscopic factors for the 1^+ state are then in good agreement, while the larger predicted value for 3^+ states is considerably less than the value quoted for the 2.92 MeV level.

There are eight model levels predicted between 3 and 4 MeV excitation energy. Of the remaining observed levels in this region, the 3.53 and 3.81 MeV levels are assigned 0^+ by (t, p) . The other levels are unassigned or ambiguous. The 3.53 MeV 0^+ is strongly excited in (t, p) , and on this basis should be assumed to arise from a configuration which violates the $N=50$ shell closure. (We note that this result implies that this sort of two-neutron-hole, two-neutron-particle state appears at 1 MeV higher in excitation energy in ^{86}K than in ^{84}Se .) However, this presumption is made ambiguous by the facts that a level at 3.53 MeV is excited with a significant $l=1$ spectroscopic factor in proton pickup and that the model predicts a third 0^+ state at 3.2 MeV, for which no experimental correspondent, in lieu of the 3.53 MeV level, would be available, unless the 3.2 MeV model state were to be pushed up to match the 3.81 MeV observed 0^+ . On the basis of the l values assigned from proton pickup, we associate the observed 3.78 MeV levels with the 1_2^+ MeV model state. We match the observed 3.94 MeV state with the 3.92 MeV model 5_1^- . This leaves approximately the same numbers of model and experimental levels unattached in this region, as seen from Table VII.

In conclusion, the numbers of model and observed levels below 4 MeV in ^{86}Kr seem in reasonable accord, both in totality and when broken down by specific spin values. If the 3.1 MeV collective 3^- level and the strongly excited 0^+ and 2^+ levels at 3.53 MeV and 4.11 MeV, respectively, are discarded, there are a few more model levels predicted than observed. If our assigned correspondences are correct, several states exhibit significant discrepancies in their predicted excitation energies, however. The predicted spectroscopic factors are in good accord overall with the data, the largest discrepancy appearing to reside in the 3_1^+ state. A particularly interesting structural feature, the spreading of the $l=1$ strength almost equally over three low-lying 2^+ states, is nicely predicted. A secure and accurate experimental census of the experimental levels below 4.5 MeV would facilitate further understanding of the structure of ^{86}Kr .

I. Levels of ^{87}Rb

The calculated and experimentally assigned energy levels of ^{87}Rb are presented in Table VIII. The level structure of ^{87}Rb has been studied experimentally with inelastic proton scattering,¹⁸ with observations of the β decay¹⁹ of the $\frac{5}{2}^+$ ground state of ^{87}Kr followed by γ decay,²⁰ with single-proton stripping²¹ and pickup²²⁻²⁴ reactions, and with neutron inelastic scattering followed by observation of the deexcitation γ rays.²⁵ Our guiding assumption in interpreting these various results is that the inelastic proton scattering study reports all existing levels and no spurious levels. The (p, p') reaction mechanism is relatively unselective and the experiment in question was performed with excellent resolution and counting statis-

tics. The (p, p') results identify every level seen in the more selective β - γ results²⁰ with the exception of eliminating a questionable¹⁹ level at 1349 keV. They similarly identify every level seen below 3.4 MeV in the $(^3\text{He}, d)$ results²¹ with the exception of a weak level reported at 1893 keV. Likewise, the levels identified²⁵ in the (n, n', γ) up through the 1741 keV level are in the (p, p') spectrum, but not the less securely placed levels reported at 1950 and 1999 keV.

The (p, p') spectrum indicates 23 levels up to 3.40 MeV excitation energy. (If we were to add the 1893 keV, $l=1$, level assigned²¹ in $(^3\text{He}, d)$ —it is not listed in Table VIII—there would be 24 levels instead.) The predicted spectrum also has 24 levels below 3.4 MeV. One of the experimentally noted levels, at 3.06 MeV excitation energy, has $J=\frac{1}{2}^+$ on the basis²¹ of an $l=0$ stripping pattern from $(^3\text{He}, d)$. Since the lowest predicted $\frac{1}{2}^+$ level has an excitation energy of 5.1 MeV, this 3.06 MeV level must be an intruder. Two model levels (the $\frac{9}{2}^-$ at 1957 keV and the $\frac{1}{2}^-$ at 3076 keV) cannot be associated with the 22 remaining (p, p') levels without violating the spin-parity assignments (limits) inferred from the (p, p') angular distributions. Other than these levels (and the 1893 keV level) there is a complete and reasonably consistent correlation of predicted and observed levels up through 3.4 MeV excitation for ^{87}Rb , as is shown in Table VIII.

The ground and first excited states of ^{87}Rb are closely related to the corresponding levels in ^{85}Br . In ^{87}Rb we have experimental results from both proton stripping and pickup. These reactions show that these two states are to a good approximation one-quasihole states relative to the $(f_{5/2})^6(p_{3/2})^4$ configuration. The stripping data for higher-lying states establish that the 1578 keV level is the $g_{9/2}$ one-quasiparticle state and suggest that this strength is more concentrated into this single state than is predicted. The other states experimentally observed in the stripping reaction with $l=1$ have small spectroscopic factors, consistent with predictions, and are useful in establishing spin-parity limits. The pickup data²²⁻²⁴ are in ways more informative, since they reveal significant admixtures of $p_{1/2}$ and $g_{9/2}$ components into the ^{88}Sr ground state. The model predictions are in agreement with the $p_{1/2}$ strength, but seriously underestimate the quoted $g_{9/2}$ strength. Other data also suggest² that the model fails to predict enough $g_{9/2}$ occupancy in ^{88}Sr .

III. CONCLUSIONS

The evaluation of our model results for the $N=50$ nuclei from ^{80}Zn to ^{87}Rb yields conclusions which vary from nucleus to nucleus. Perhaps the only constants are the need to have more, and more precise, experimental data and the need to have better quantitative accuracy in the predicted level energies.

The degree of completeness with which the model spectra account for measured spectra depends upon the nucleus studied. For the lightest, least thoroughly explored nuclei, the model produces as many, or more, levels than are observed, but there are obvious questions about how many more levels remain to be discovered. For ^{84}Se , there is clear evidence for low-lying two-

neutron-hole, two-neutron-particle intruders in the experimental spectrum. The experimental knowledge of the spectra of ^{85}Br , ^{86}Kr , and ^{87}Rb is progressively more complete with increasing mass, and the correspondences between model and measured levels become correspondingly more complete and accurate, with the observed spectra of ^{86}Kr and ^{87}Rb being reproduced fairly completely up to 4 and 3 MeV of excitation energy, respectively, albeit with some unacceptably large deviations in energy for some levels. It is clear that knowledge of existing levels, along with their spins, parities and decay characteristics, must be improved for substantial progress in the current level of theoretical understanding to be possible. Some of the conclusions reached in this analysis already suggest alterations in the assumptions made about the level-energy data used¹ to generate the model Hamiltonian. Ultimately, an additional cycle in the iteration of Hamiltonian two-way matrix elements to fit the (corrected and expanded) set of level-energy data will be appropriate.

Analysis of predicted and experimentally based spectroscopic factors for ^{85}Br , ^{86}Kr , and ^{87}Rb shows that the makeup of the low-lying model states in terms of $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ protons is consistent with experimental observation. The $f_{5/2}$ orbit fills first as the number of protons increases above $Z=28$, with the $p_{3/2}$ orbit being the principal source of configuration admixtures. The $p_{1/2}$ and $g_{9/2}$ orbits are well separated from the $f_{5/2}$ and $p_{3/2}$ in the lighter isotones. For ^{83}As , ^{85}Br ,

and ^{87}Rb , the lowest pair of states are the $p_{3/2}$ and $f_{5/2}$ single quasiparticle states. The $p_{1/2}$ and $g_{9/2}$ single-quasiparticle states are not clearly identified experimentally until ^{87}Rb , where they occur at 0.8 and 1.5 MeV excitation, respectively. They presumably are descending in excitation energy rapidly from ^{85}Br where they are predicted to occur at 1.2 and 2.0 MeV, respectively. The relative admixtures of $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ configurations in the ground states of ^{86}Kr , ^{87}Rb , and ^{88}Sr as revealed by pickup experiments is consistent with the model predictions. However, there is a clear suggestion that the amount of predicted $g_{9/2}$ occupation in ^{86}K and ^{88}Sr is significantly less than the extracted experimental spectroscopic factors suggest. While these data are somewhat tenuous, the implied problem with the model wave function echos independent suggestions² that the model ^{88}Sr ground state wave function contains too little $g_{9/2}$ occupancy.

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